A Polynomial Spilling Heuristic: Layered Allocation

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Register Allocation

The register allocation problem maps temporary variables to machine registers

The Allocation/Spilling Problem

- The allocation chooses the register residents
- It also aims at minimizing the load/store overhead

Assignment/Coloring

- The coloring decides which register is used by which variable

Decoupling

- For the moment, let us assume that these two problems can be decoupled

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A bit of Terminology

- **Maxlive**: the maximum number of simultaneously live variables
- Given V a set of variables of a program and R a number of available registers

Two sub-problems

- The **lowering problem** finds S, a subset of V, of minimum cost to spill in order to decrease maxlive by a small number
- The **single layer allocation problem** finds A, a subset of V, of maximum cost to allocate to a small number of registers

Two Approaches to the Allocation Problem

- The **layered allocation** incrementally solves the single layer allocation problem until the sum of the used registers reaches R
- The **incremental lowering** incrementally solves the lowering problem until maxlive reaches R

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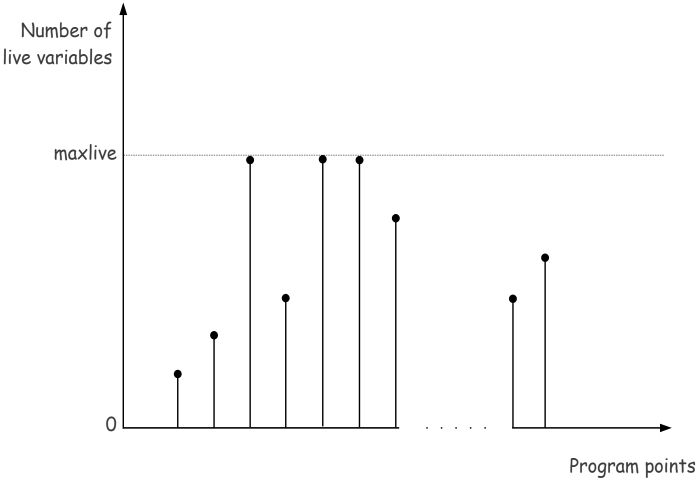
Lowering vs. Layering



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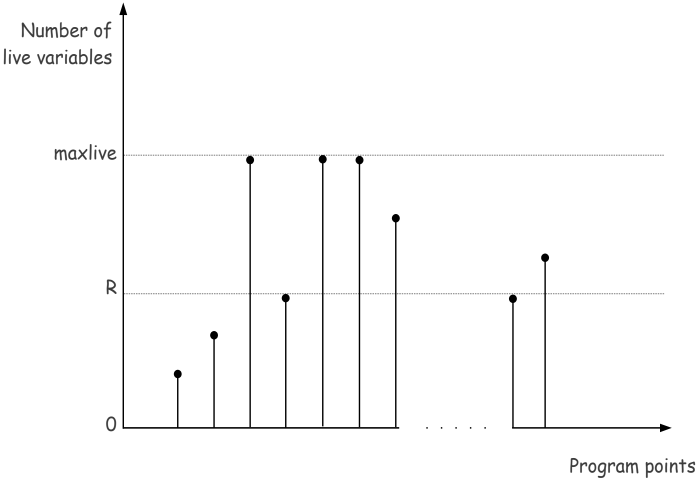

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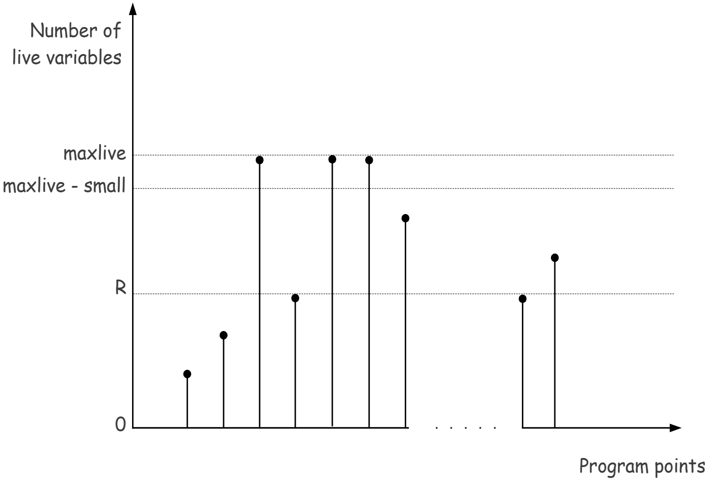

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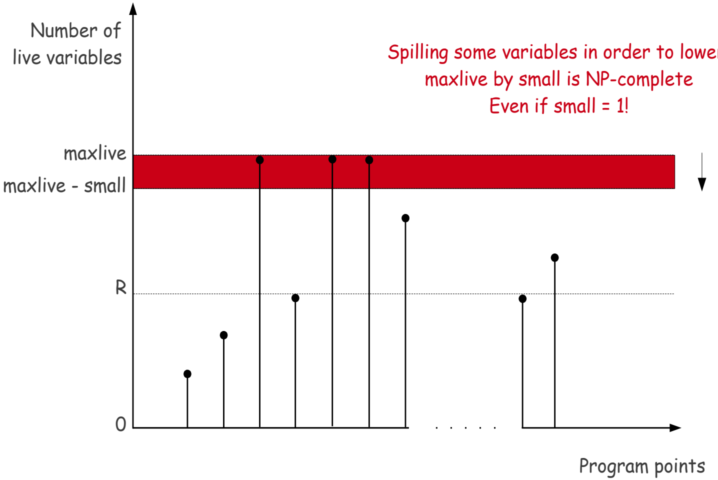

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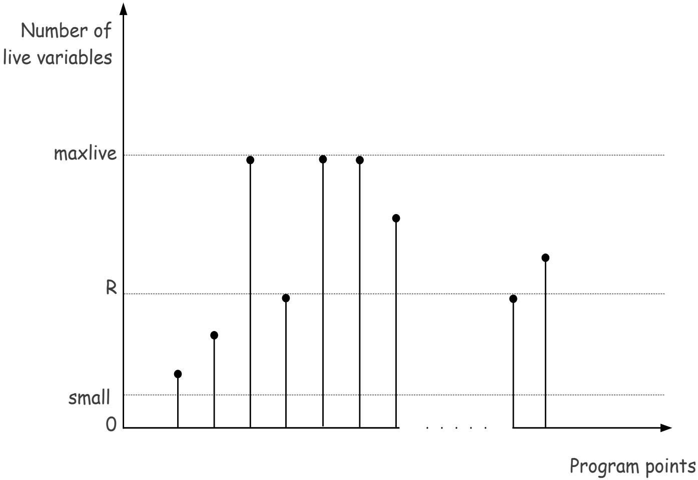

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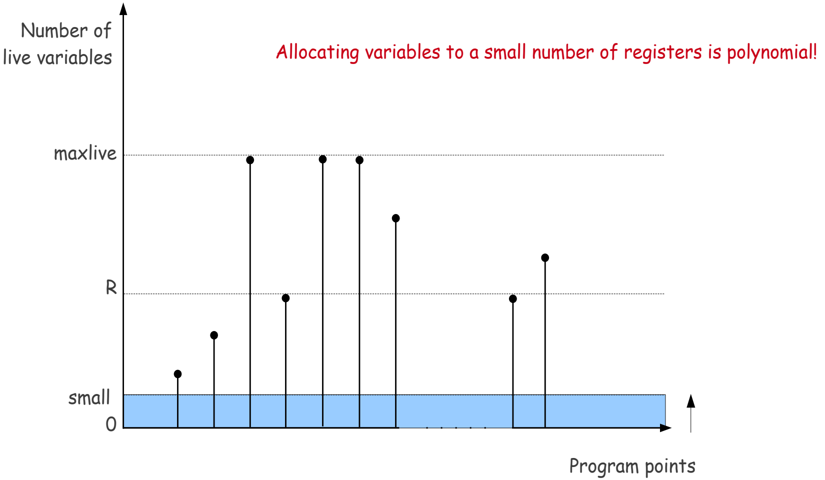

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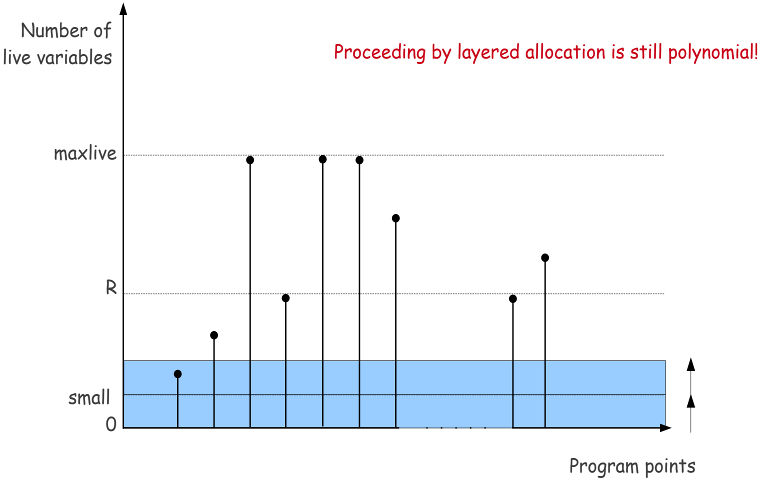

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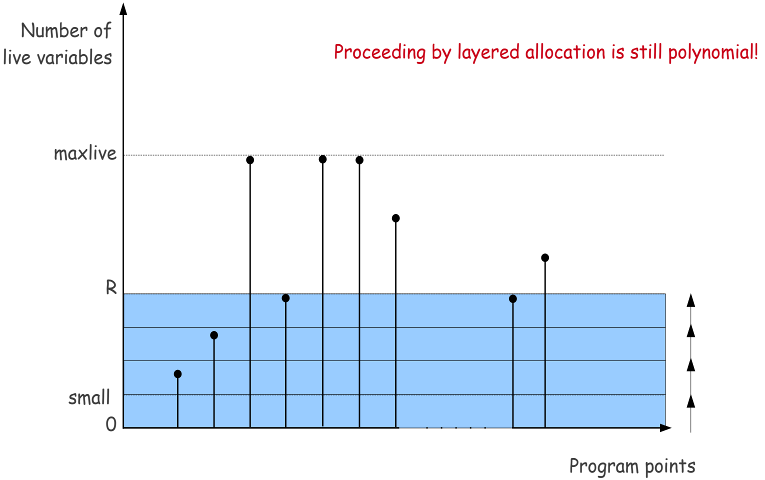

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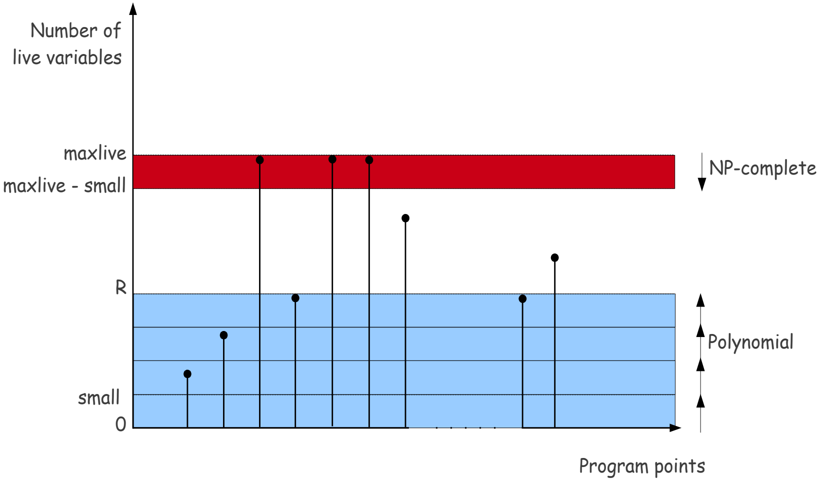

Why should the Layered Allocation be Close to Optimal?

- Let us assume that we have a program P
- When $R+1$ registers are available, let us call $SPILL^P_{R+1}$ the optimal set of variables to spill to make a coloring possible
- For most programs, $SPILL^P_{R+1} \subset SPILL^P_R$ **[Diouf'10]**
- Hence, for most programs, $ALLOC^P_R \subset ALLOC^P_{R+1}$

Taxonomy of the Approaches

Approach	Complexity	Quality
Allocation/Spilling	NP-complete	Optimal
Layered Allocation	Polynomial	Close to optimal
Incremental lowering-optimal	NP-complete	???
Incremental lowering-heuristic	Polynomial	Not-optimal

To make it clear!

- The Allocation problem is NP-complete
- The Layered allocation is a heuristic that is close to optimal allocation
- We are not turning an NP-complete problem into a polynomial one

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Outline

Introduction

Layered Approach

Layered-Heuristic Allocation: General Graphs

Layered-Optimal Allocation: Chordal Graphs

Experimental Evaluation

Conclusion

Two Heuristics for the Spilling Problem

Input:

1. A register allocation problem where each variable has an estimated spill cost
2. A number of available registers

Objective:

We want to perform an allocation that minimizes the cost of all the spilled variables

Two graph-based solutions :

- The general approach: Layered-Heuristic Register Allocator
- The SSA-based approach: Layered-Optimal Register Allocator

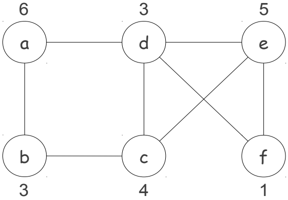
The General Approach

Given an interference graph of a program and `R` available registers (colors)

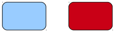
1. Assume that we have `one` register
2. We approximate the set of nodes of maximum cost/weight to allocate with *one* register: a layer. This layer is an independent set.
3. Remove the nodes of the layer from the graph at the next iteration

Repeat these instructions until we reach `R` or we allocate all the variables

How the Layered-Heuristic Works



2 available registers



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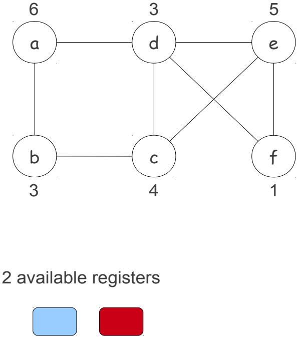
Variables sorted by decreasing cost: a, e, c, b, d, f



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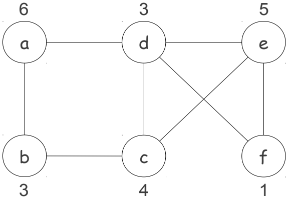

I-Set-1: {a

2 available registers

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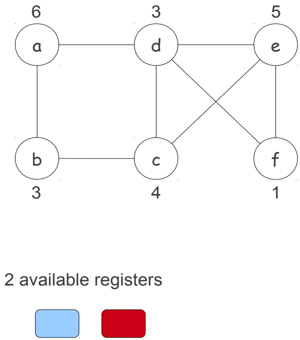
Variables sorted by decreasing cost: e, c, b, d, f



I-Set-1: {a,e}

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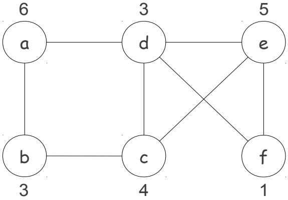
Variables sorted by decreasing cost: c, b, d, f



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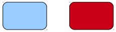
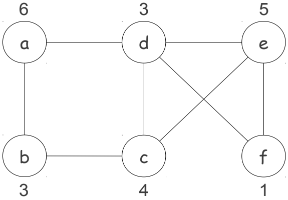


How the Layered-Heuristic Works

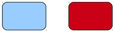
Variables sorted by decreasing cost: b, d, f



I-Set-1: {a,e}

I-Set-2: {c,

2 available registers



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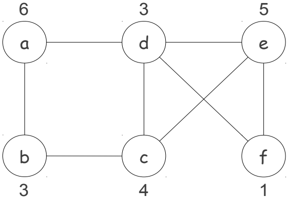
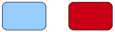
Variables sorted by decreasing cost: b, d



I-Set-1: {a,e}

I-Set-2: {c,f}

2 available registers



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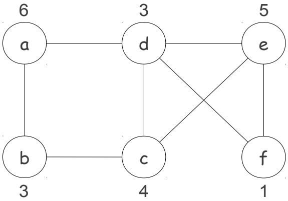

I-Set-1: {a,e}

I-Set-2: {c,f}

I-Set-3: {b,d}

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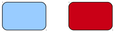

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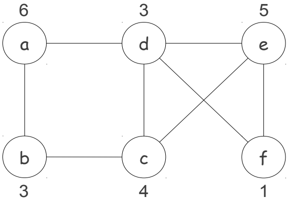

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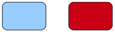
I-Sets sorted by decreasing cost: I-Set-1, I-Set-3, I-Set-2

2 available registers



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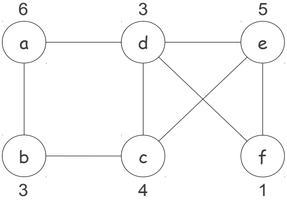

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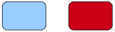

The cost of the allocation is **5**

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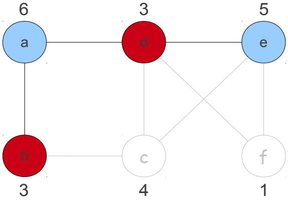

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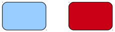

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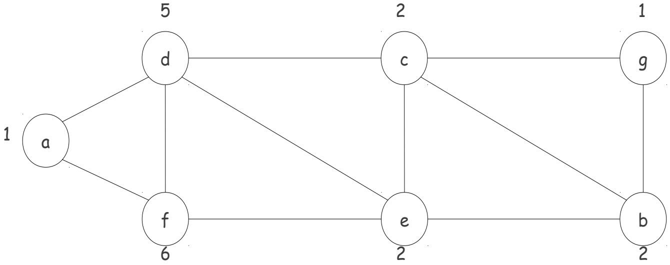
SSA-based Interference Graphs

The interference graph of an SSA-based program is chordal

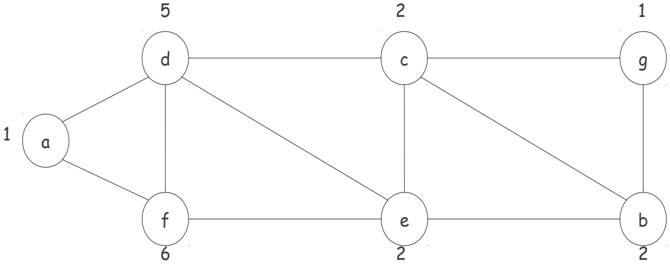
1. The allocation problem can be decoupled from the coloring problem thanks to maxlive
2. Hence, the maximum weighted independent set can be found optimally **[Frank'75]**

The Maximum Weighted Independent Set Algorithm



Weighted graph

The Maximum Weighted Independent Set Algorithm



Weighted graph

iteration	a	f	d	e	b	g	c	red vertices
-	1	6	5	2	2	1	2	Ø

Red vertices

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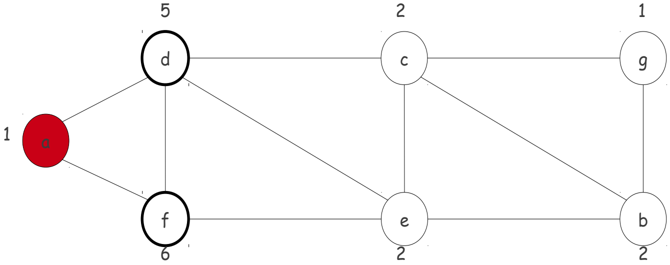

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1		5	4	2	2	1	2	a

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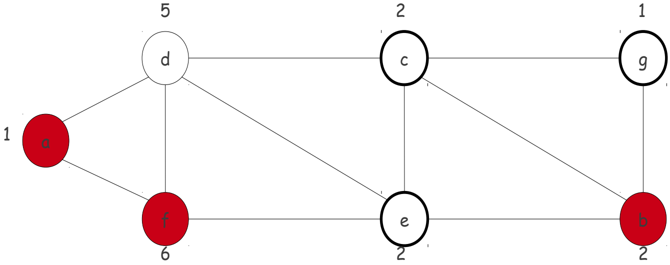

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5						*-1*	0	b,f,a

Red vertices

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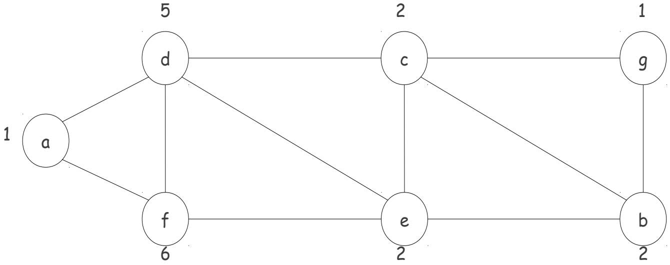

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Red vertices

iteration	red vertices	blue vertices
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Blue vertices

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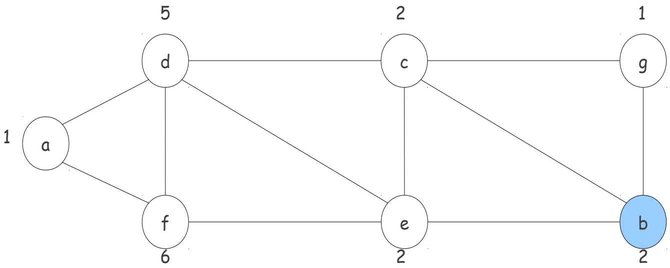

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1	f,a	b

Blue vertices

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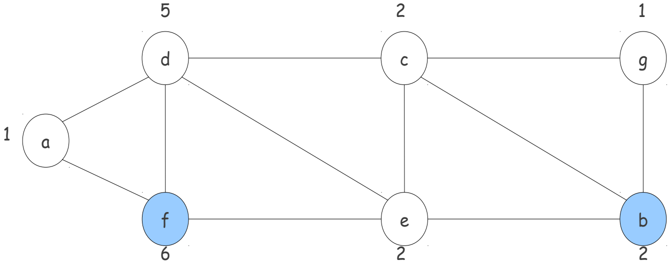

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Red vertices

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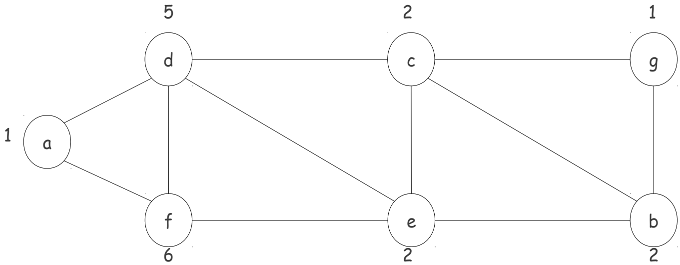
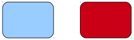
-	b,f,a	Ø
1	f,a	b
2	Ø	b, f

Blue vertices

How the Layered-Optimal Allocator Works



2 available registers



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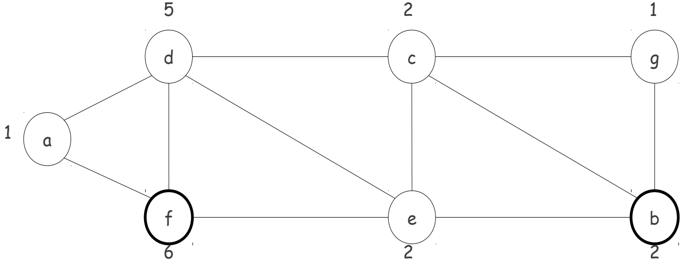

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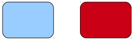

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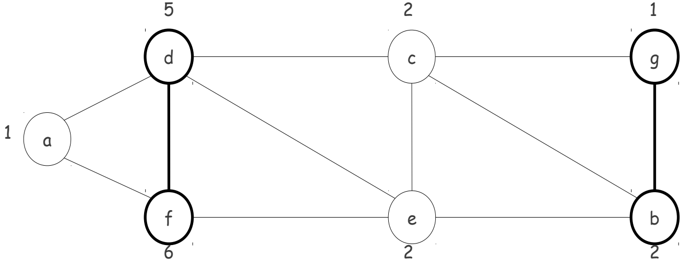

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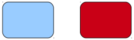

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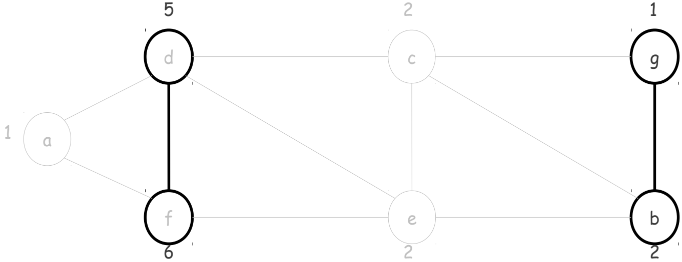

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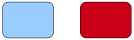
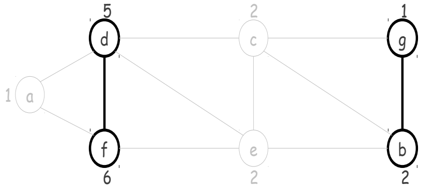
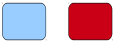


A First Improvement: Weights Bias

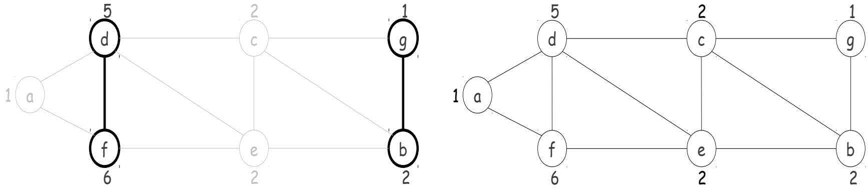


Allocated variables: {f, b, d, g}
Allocation-Cost = 14

2 available registers



A First Improvement: Weights Bias

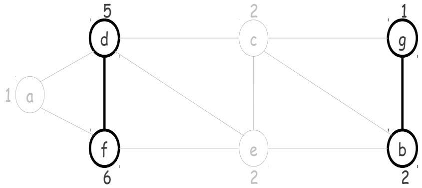


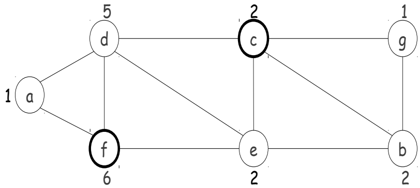
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

Allocated variables: { }

2 available registers

A First Improvement: Weights Bias



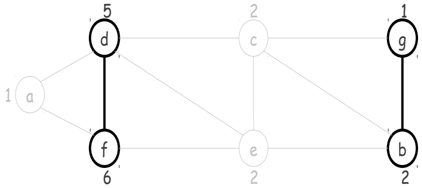


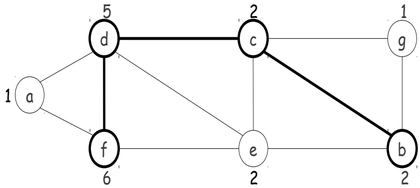
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

Allocated variables: {f, c}

2 available registers

A First Improvement: Weights Bias

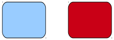




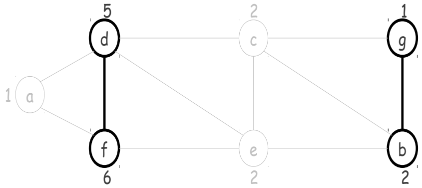
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

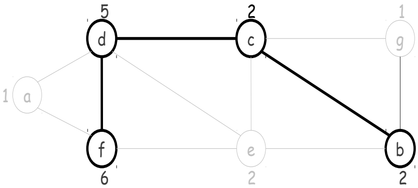
Allocated variables: {f, c, d, b}

2 available registers



A First Improvement: Weights Bias

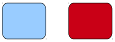




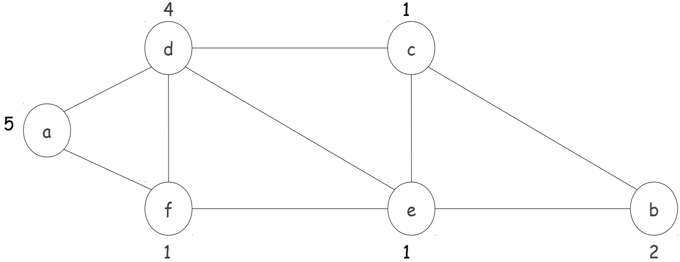
Allocated variables: {f, b, d, g}
Allocation-Cost = 14

Allocated variables: {f, c, d, b}
Allocation-Cost = 15

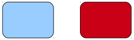
2 available registers



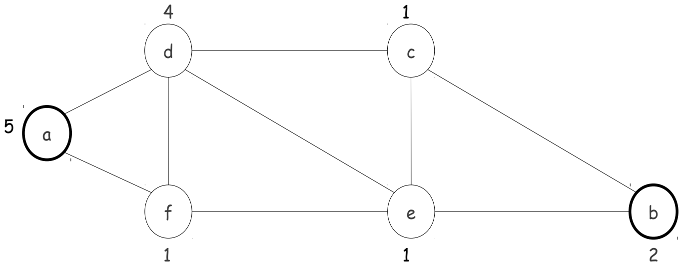
A Second Improvement: A Fixed Point Iteration



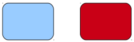
2 available registers



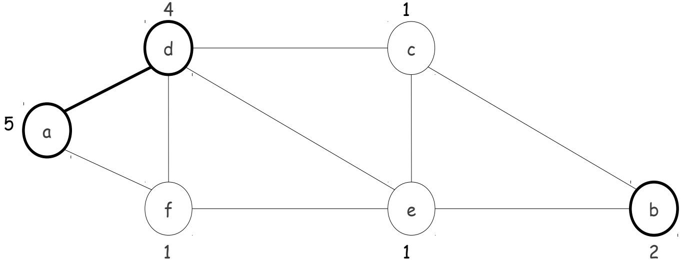
A Second Improvement: A Fixed Point Iteration



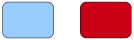
2 available registers



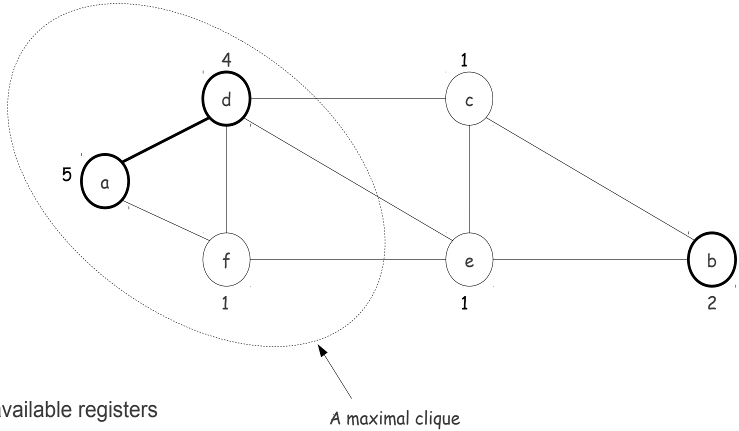
A Second Improvement: A Fixed Point Iteration



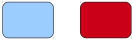
2 available registers



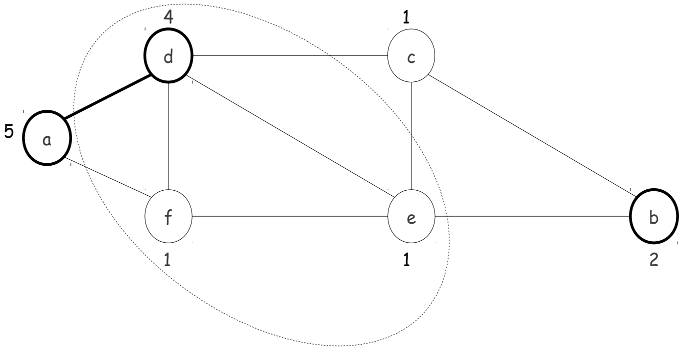
A Second Improvement: A Fixed Point Iteration



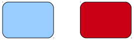
2 available registers



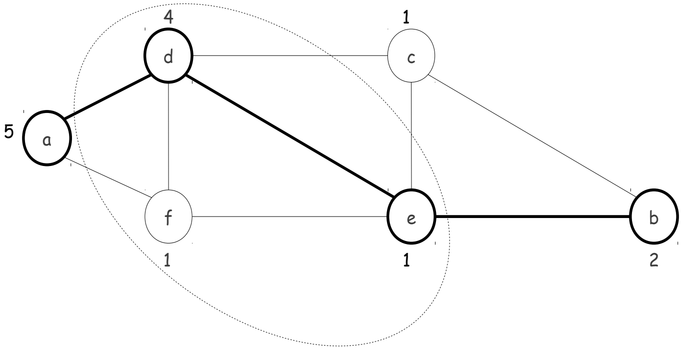
A Second Improvement: A Fixed Point Iteration



2 available registers

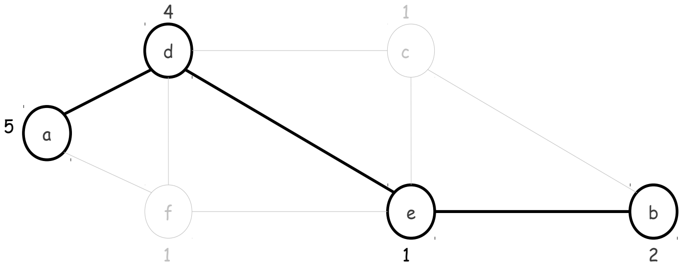


A Second Improvement: A Fixed Point Iteration



2 available registers

A Second Improvement: A Fixed Point Iteration



2 available registers

Outline

Introduction

Layered Approach
- Layered-Heuristic Allocation: General Graphs
- Layered-Optimal Allocation: Chordal Graphs

Experimental Evaluation

Conclusion

Evaluating the Layered-Heuristic Allocator

Architectures

- x86

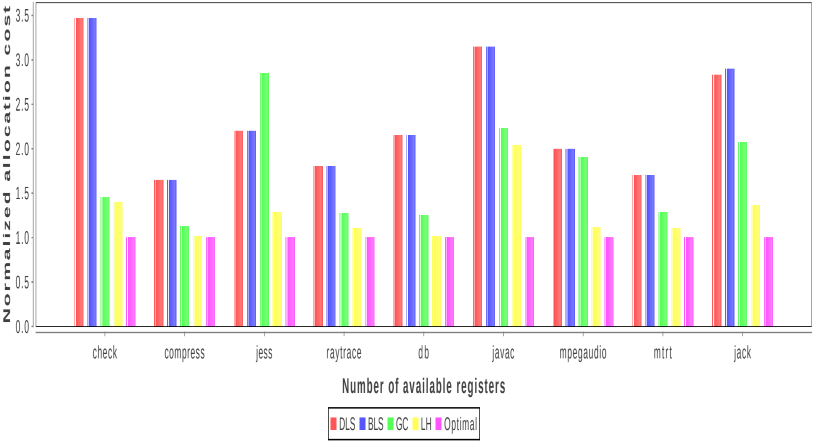
Benchmarks extracted from JikesRVM

- SPEC JVM 98

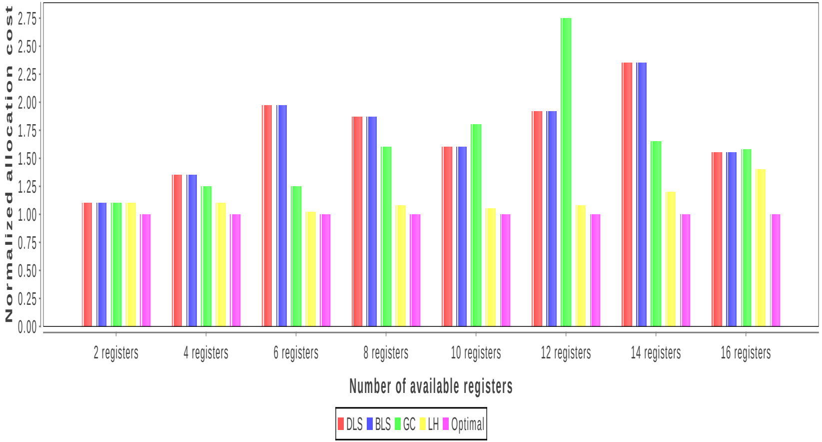
Algorithms

- LS: the linear scan implemented in JikesRVM
- BLS: a variant of the Belady's furthest -first
- GC: the Chaitin-Briggs optimistic graph coloring
- Optimal: an ILP-based Allocator
- LH: Layered Heuristic

Evaluating the Layered-Heuristic Allocator



Evaluating the Layered-Heuristic Allocator



Evaluating the Layered-Optimal Allocator

Architectures

- ARMv7
- ST231

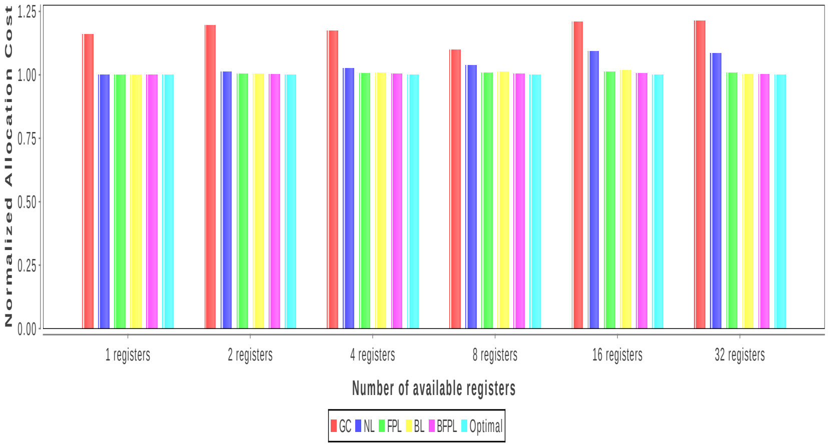
Benchmarks

- eembc
- lao-kernels
- SPEC CPU 2000int

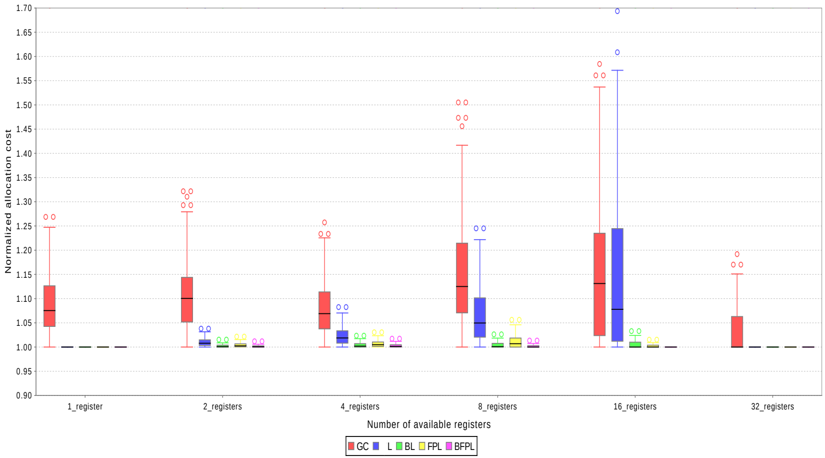
Algorithms

- GC: the Chaitin-Briggs optimistic graph coloring
- Optimal: an ILP-based Allocator
- L: our baseline Layered-Optimal approach
- BL: the biased variant of our Layered-Optimal
- FPL: the fixed-point variant of our Layered-Optimal
- BFPL: the biased and fixed-point variant of our Layered-Optimal

Evaluating the Layered-Optimal Allocator



Evaluating the Layered-Optimal Allocator



Outline

Introduction

Layered Approach

Layered-Heuristic Allocation: General Graphs

Layered-Optimal Allocation: Chordal Graphs

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Conclusion

Contributions

- Layered allocation: polynomial and close to optimal allocation
- Iteratively allocate instead of (classical) iteratively spilling
- The approach works on general graphs and on SSA-based graphs